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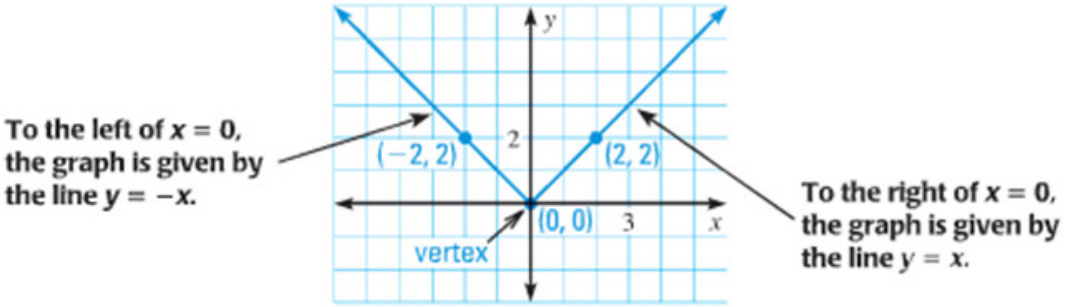
Absolute Value

Parent Function for Absolute Value Functions

The parent function for the family of all absolute value functions is $f(x) = |x|$. The graph of $f(x) = |x|$ is V-shaped and is symmetric about the y-axis. So, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.



The highest or lowest point on the graph of an absolute value function is called the **vertex**. The vertex of the graph of $f(x) = |x|$ is $(0, 0)$.

vertex: (0,0)

$y = |x|$

right 2

$y = |x - 2| + 4$

up 4

vertex: (2,4)

left 3

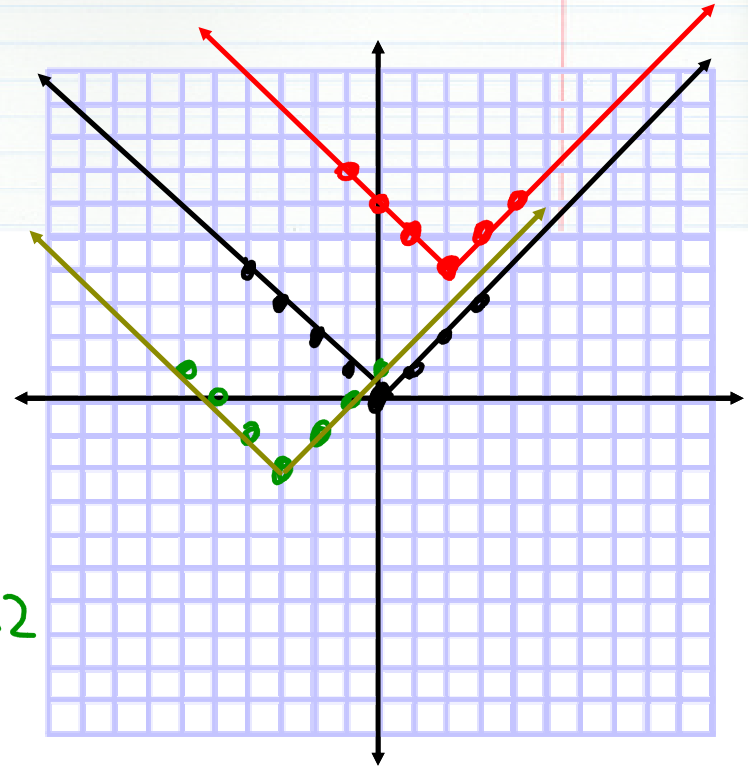
vertex: (-3,-2)

$x = -3$

$y = |x + 3| - 2$

down 2

$y = |x - h| + k$



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Transformations of General Graphs

The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of any function $y = f(x)$ by performing these steps:

STEP 1 **Stretch or shrink** the graph of $y = f(x)$ vertically by a factor of $|a|$ if $|a| \neq 1$. If $|a| > 1$, stretch the graph. If $|a| < 1$, shrink the graph.

STEP 2 **Reflect** the resulting graph from Step 1 in the x-axis if $a < 0$.

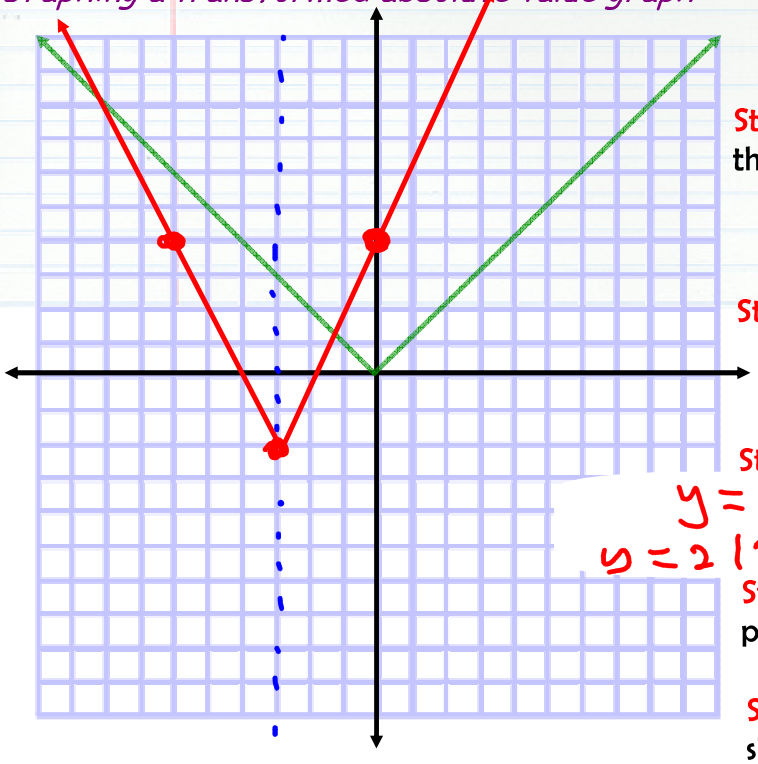
STEP 3 **Translate** the resulting graph from Step 2 horizontally h units and vertically k units.

STRETCHES, SHRINKS, AND REFLECTIONS When $|a| \neq 1$, the graph of $y = a|x|$ is a vertical *stretch* or a vertical *shrink* of the graph of $y = |x|$, depending on whether $|a|$ is less than or greater than 1.

For $\|a\| > 1$	For $\|a\| < 1$
• The graph is vertically *stretched*, or elongated.	• The graph is vertically *shrunk*, or compressed.
• The graph of $y = a\|x\|$ is *narrower* than the graph of $y = \|x\|$.	• The graph of $y = a\|x\|$ is *wider* than the graph of $y = \|x\|$.

When $a = -1$, the graph of $y = a|x|$ is a **reflection** in the x-axis of the graph of $y = |x|$. When $a < 0$ but $a \neq -1$, the graph of $y = a|x|$ is a vertical stretch or shrink with a reflection in the x-axis of the graph of $y = |x|$.

Graphing a transformed absolute value graph:



$|x - h|$

$y = 2|x + 3| - 2$

Step 0: Get a general idea of what the graph will look like.

✓ left 3, down 2
steeper/narrower

Step 1: Identify and plot the vertex.

$(-3, -2)$

Step 2: Plot another point, such as (0, ?)

$y = 2|0 + 3| - 2$
$y = 2|3| - 2 = 6 - 2 = 4$ $(0, 4)$

Step 3: Use symmetry to plot a 3rd point.

Step 4: Connect the points with a v-shaped graph.